12. $f_1(x)g_1(y) + f_2(x)g_2(y) + f_3(x)g_3(y) = 0.$

Bilinear functional equation - 1.

Two solutions:

$$f_1(x) = C_1 f_3(x), \quad f_2(x) = C_2 f_3(x), \quad g_3(y) = -C_1 g_1(y) - C_2 g_2(y);$$
$$g_1(y) = C_1 g_3(y), \quad g_2(y) = C_2 g_3(y), \quad f_3(x) = -C_1 f_1(x) - C_2 f_2(x),$$

where C_1 and C_2 are arbitrary constants, the functions on the right-hand sides of the solutions are arbitrary.

Reference

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Nonlinear Partial Differential Equations (Supplement S.4.4)*, Chapman & Hall/CRC Press, Boca Raton, 2004.

