**13.** $f_1(x)g_1(y) + f_2(x)g_2(y) + f_3(x)g_3(y) + f_4(x)g_4(y) = 0.$

***Bilinear functional equation - 2.***

Equations of this type often arise in generalized separation of variables in nonlinear PDEs.

1°. Solution:

$$f_1(x) = C_1 f_3(x) + C_2 f_4(x), \qquad f_2(x) = C_3 f_3(x) + C_4 f_4(x),$$
$$g_3(y) = -C_1 g_1(y) - C_3 g_2(y), \qquad g_4(y) = -C_2 g_1(y) - C_4 g_2(y)$$

dependent on four arbitrary constants $C_1, \dots, C_4$. The functions on the right-hand sides of the solution are arbitrary.

2°. The equation has also two other solutions:

$$f_1(x) = C_1 f_4(x), \quad f_2(x) = C_2 f_4(x), \quad f_3(x) = C_3 f_4(x), \quad g_4(y) = -C_1 g_1(y) - C_2 g_2(y) - C_3 g_3(y);$$
$$g_1(y) = C_1 g_4(y), \quad g_2(y) = C_2 g_4(y), \quad g_3(y) = C_3 g_4(y), \quad f_4(x) = -C_1 f_1(x) - C_2 f_2(x) - C_3 f_3(x)$$

involving three arbitrary constants.

## Reference

**Polyanin, A. D. and Zaitsev, V. F.,** *Handbook of Nonlinear Partial Differential Equations (Supplement S.4.4)*, Chapman & Hall/CRC Press, Boca Raton, 2004.



 