



First-Order Partial Differential Equations > Linear Equations > Section 1.2

8.
$$\frac{\partial w}{\partial x} + [ay + f(x)] \frac{\partial w}{\partial y} = g(x)h(y).$$

General solution:

$$w = \int g(x) h \left(e^{ax} u + e^{ax} \int f(x) e^{-ax} dx \right) dx + \Phi(u), \quad \text{where } u = e^{-ax} y - \int f(x) e^{-ax} dx.$$

Here, $\Phi(u)$ is an arbitrary function; in the integration, u is considered a parameter.

Reference

Polyanin, A. D., Zaitsev, V. F., and Moussiaux, A., *Handbook of First Order Partial Differential Equations*, Taylor & Francis, London, 2002.