12. $f(x)\dfrac{\partial w}{\partial x} + g(y)\dfrac{\partial w}{\partial y} = h_1(x) + h_2(y).$

General solution:

$$w = \int \frac{h_1(x)}{f(x)}\,dx + \int \frac{h_2(y)}{g(y)}\,dy + \Phi\left(\int \frac{dx}{f(x)} - \int \frac{dy}{g(y)}\right),$$

where $\Phi(u)$ is an arbitrary function.

Reference

Polyanin, A. D., Zaitsev, V. F., and Moussiaux, A., *Handbook of First Order Partial Differential Equations*, Taylor & Francis, London, 2002.

