



First-Order Partial Differential Equations > Linear Equations > Section 1.3

5.
$$\frac{\partial w}{\partial x} + a \frac{\partial w}{\partial y} = f(x, y)w + g(x, y).$$

General solution:

$$w = F(x, u) \left[\Phi(u) + \int \frac{g(x, u + ax)}{F(x, u)} dx \right], \quad F(x, u) = \exp \left[\int f(x, u + ax) dx \right],$$

where $u = y - ax$ and $\Phi(u)$ is an arbitrary function. In the integration, u is considered a parameter.

Reference

Polyanin, A. D., Zaitsev, V. F., and Moussiaux, A., *Handbook of First Order Partial Differential Equations*, Taylor & Francis, London, 2002.