



First-Order Partial Differential Equations > Linear Equations > Section 1.3

$$6. \quad ax \frac{\partial w}{\partial x} + by \frac{\partial w}{\partial y} = f(x)w + g(x).$$

General solution:

$$w = \exp \left[\frac{1}{a} \int \frac{f(x) dx}{x} \right] \left\{ \Phi(x^{-b/a}y) + \frac{1}{a} \int \frac{g(x)}{x} \exp \left[-\frac{1}{a} \int \frac{f(x) dx}{x} \right] dx \right\},$$

where $\Phi(u)$ is an arbitrary function.

Reference

Polyanin, A. D., Zaitsev, V. F., and Moussiaux, A., *Handbook of First Order Partial Differential Equations*, Taylor & Francis, London, 2002.