



First-Order Partial Differential Equations > Linear Equations > Section 1.3

$$10. \quad f_1(x) \frac{\partial w}{\partial x} + f_2(y) \frac{\partial w}{\partial y} = aw + g_1(x) + g_2(y).$$

General solution:

$$w = E_1(x)\Phi(u) + E_1(x) \int \frac{g_1(x) dx}{f_1(x)E_1(x)} + E_2(y) \int \frac{g_2(y) dy}{f_2(y)E_2(y)},$$

where

$$E_1(x) = \exp \left[a \int \frac{dx}{f_1(x)} \right], \quad E_2(y) = \exp \left[a \int \frac{dy}{f_2(y)} \right], \quad u = \int \frac{dx}{f_1(x)} - \int \frac{dy}{f_2(y)},$$

$\Phi(u)$ is an arbitrary function.

Reference

Polyanin, A. D., Zaitsev, V. F., and Moussiaux, A., *Handbook of First Order Partial Differential Equations*, Taylor & Francis, London, 2002.