



First-Order Partial Differential Equations > Quasilinear Equations > Section 2.3

5. $\frac{\partial w}{\partial x} + f(w) \frac{\partial w}{\partial y} = g(y).$

General solution:

$$x = \int_{y_0}^y \psi(G(t) - G(y) + F(w)) dt + \Phi(F(w) - G(y)),$$

where $G(y) = \int g(y) dy$, $F(w) = \int f(w) dw$, and $\Phi(u)$ is an arbitrary function. The function $\psi = \psi(z)$ is defined parametrically by $\psi = \frac{1}{f(w)}$, $z = F(w)$.

Reference

Polyanin, A. D., Zaitsev, V. F., and Moussiaux, A., *Handbook of First Order Partial Differential Equations*, Taylor & Francis, London, 2002.