2. $\dfrac{\partial w}{\partial x} + a\left(\dfrac{\partial w}{\partial y}\right)^2 + by^2 = 0.$

This equation governs free oscillations of a point body of mass $m = 1/(2a)$ in an elastic field with elastic coefficient $k = 2b$ (x is time and y is the displacement from the equilibrium).

Complete integral:

$$w = -C_1 x + C_2 \pm \int \sqrt{\frac{C_1 - by^2}{a}}\, dx + C_2,$$

where C_1 and C_2 are arbitrary constants.

References

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