1. $a\left(\frac{\partial w}{\partial x}\right)^2 + b\left(\frac{\partial w}{\partial y}\right)^2 = c.$

For $a = b$, this is a differential equation of light rays.

1°. Complete integral:

$$w = C_1 x + C_2 y + C_3, \quad \text{where} \quad aC_1^2 + bC_2^2 = c,$$

C_1 and C_3 are arbitrary constants.

2°. An alternative form of the complete integral:

$$\frac{w^2}{c} = \frac{(x - C_1)^2}{a} + \frac{(y - C_2)^2}{b},$$

where C_1 and C_2 are arbitrary constants.

References

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