



First-Order Partial Differential Equations > Nonlinear Equations > Section 3.2

$$3. \left(\frac{\partial w}{\partial x} \right)^2 + \left(\frac{\partial w}{\partial y} \right)^2 = \frac{a}{\sqrt{x^2 + y^2}} + b.$$

This equation arises from the solution of the two-body problem in celestial mechanics.

Complete integral:

$$w = \pm \int \sqrt{b + \frac{a}{r} - \frac{C_1^2}{r^2}} dr + C_1 \arctan \frac{y}{x} + C_2, \quad \text{where } r = \sqrt{x^2 + y^2},$$

C_1 and C_2 are arbitrary constants.

References

- Appell, P.**, *Traité de Mécanique Rationnelle, T. 1: Statique. Dynamique du Point (Ed. 6)*, Gauthier-Villars, Paris, 1953.
- Kamke, E.**, *Differentialgleichungen: Lösungsmethoden und Lösungen, II, Partielle Differentialgleichungen Erster Ordnung für eine gesuchte Funktion*, Akad. Verlagsgesellschaft Geest & Portig, Leipzig, 1965.
- Polyanin, A. D., Zaitsev, V. F., and Moussiaux, A.**, *Handbook of First Order Partial Differential Equations*, Taylor & Francis, London, 2002.