6. $\left(\dfrac{\partial w}{\partial x}\right)^2 + \left(\dfrac{\partial w}{\partial y}\right)^2 = f(x^2 + y^2).$

Hamilton's equation for the plane motion of a point mass under the action of a central force.

Complete integral:

$$w = C_1 \arctan\frac{x}{y} + C_2 \pm \frac{1}{2}\int \sqrt{zf(z) - C_1^2}\,\frac{dz}{z}, \qquad z = x^2 + y^2,$$

where C_1 and C_2 are arbitrary constants.

References

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