



First-Order Partial Differential Equations > Nonlinear Equations > Section 3.2

$$8. \left(\frac{\partial w}{\partial x} \right)^2 + \frac{1}{x^2} \left(\frac{\partial w}{\partial y} \right)^2 = f(x).$$

This equation governs the plane motion of a point mass in a central force field, with x and y being the polar coordinates.

Complete integral:

$$w = C_1 y \pm \int \sqrt{f(x) - \frac{C_1^2}{x^2}} dx + C_2,$$

where C_1 and C_2 are arbitrary constants.

References

- Appell, P.**, *Traité de Mécanique Rationnelle, T. 1: Statique. Dynamique du Point (Ed. 6)*, Gauthier-Villars, Paris, 1953.
Polyanin, A. D., Zaitsev, V. F., and Moussiaux, A., *Handbook of First Order Partial Differential Equations*, Taylor & Francis, London, 2002.