



$$12. \quad f_1(x) \left( \frac{\partial w}{\partial x} \right)^2 + f_2(y) \left( \frac{\partial w}{\partial y} \right)^2 = g_1(x) + g_2(y).$$

*A separable equation.* This equation is encountered in differential geometry in studying geodesic lines of Liouville surfaces.

Complete integral in implicit form:

$$w = \pm \int \sqrt{\frac{g_1(x) + C_1}{f_1(x)}} dx \pm \int \sqrt{\frac{g_2(y) - C_1}{f_2(y)}} dy + C_2,$$

where  $C_1$  and  $C_2$  are arbitrary constants. The signs before each of the integrals can be chosen independently of each other.

### References

- Appell, P.**, *Traité de Mécanique Rationnelle, T. 1: Statique. Dynamique du Point (Ed. 6)*, Gauthier-Villars, Paris, 1953.
- Kamke, E.**, *Differentialgleichungen: Lösungsmethoden und Lösungen, II, Partielle Differentialgleichungen Erster Ordnung für eine gesuchte Funktion*, Akad. Verlagsgesellschaft Geest & Portig, Leipzig, 1965.
- Polyanin, A. D., Zaitsev, V. F., and Moussiaux, A.**, *Handbook of First Order Partial Differential Equations*, Taylor & Francis, London, 2002.