



First-Order Partial Differential Equations > Nonlinear Equations > Section 3.3

$$17. \left(\frac{\partial w}{\partial x} \right)^2 + \left(\frac{\partial w}{\partial y} \right)^2 = F \left(x^2 + y^2, y \frac{\partial w}{\partial x} - x \frac{\partial w}{\partial y} \right).$$

Complete integral:

$$w = -C_1 \arctan \frac{y}{x} + \frac{1}{2} \int \sqrt{\xi F(\xi, C_1) - C_1^2} \frac{d\xi}{\xi} + C_2, \quad \text{where } \xi = x^2 + y^2,$$

C_1 and C_2 are arbitrary constants.

References

Kamke, E., *Differentialgleichungen: Lösungsmethoden und Lösungen, II, Partielle Differentialgleichungen Erster Ordnung für eine gesuchte Funktion*, Akad. Verlagsgesellschaft Geest & Portig, Leipzig, 1965.

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