



First-Order Partial Differential Equations > Nonlinear Equations > Section 3.3

$$19. \quad F\left(ax + by, \frac{\partial w}{\partial x}, \frac{\partial w}{\partial y}\right) = 0.$$

For  $b = 0$ , see equation 3.3.18.

Complete integral for  $b \neq 0$ :

$$w = C_1x + \varphi(z, C_1) + C_2, \quad z = ax + by,$$

where  $C_1$  and  $C_2$  are arbitrary constants, and the function  $\varphi = \varphi(z)$  is determined from the nonlinear ordinary differential equation  $F(z, a\varphi'_z + C_1, b\varphi'_z) = 0$ .

### Reference

**Polyanin, A. D., Zaitsev, V. F., and Moussiaux, A.,** *Handbook of First Order Partial Differential Equations*, Taylor & Francis, London, 2002.