25. $F\left(x, \dfrac{\partial w}{\partial x}, G\left(y, \dfrac{\partial w}{\partial y}\right)\right) = 0.$

Separable equation.

Complete integral:

$$w = \varphi(x, C_1) + \psi(y, C_1) + C_2,$$

where C_1 and C_2 are arbitrary constants, and the functions φ and ψ are determined by the ordinary differential equations

$$F(x, \varphi'_x, C_1) = 0, \qquad G(y, \psi'_y) = C_1.$$

On solving these equations for the derivatives, we obtain linear separable equations, which are easy to integrate.

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