1. First-Order Linear Partial Differential Equations

1.1. Equations of the Form $f(x,y)\frac{\partial w}{\partial x}+g(x,y)\frac{\partial w}{\partial y}=0$

1. $\dfrac{\partial w}{\partial x}+\big[f(x)y+g(x)\big]\dfrac{\partial w}{\partial y}=0.$

2. $\dfrac{\partial w}{\partial x}+\big[f(x)y+g(x)y^k\big]\dfrac{\partial w}{\partial y}=0.$

3. $\dfrac{\partial w}{\partial x}+\big[f(x)e^{\lambda y}+g(x)\big]\dfrac{\partial w}{\partial y}=0.$

4. $f(x)\dfrac{\partial w}{\partial x}+g(y)\dfrac{\partial w}{\partial y}=0.$

5. $\big[f(y)+amx^ny^{m-1}\big]\dfrac{\partial w}{\partial x}-\big[g(x)+anx^{n-1}y^m\big]\dfrac{\partial w}{\partial y}=0.$

6. $\big[e^{\alpha x}f(y)+c\beta\big]\dfrac{\partial w}{\partial x}-\big[e^{\beta y}g(x)+c\alpha\big]\dfrac{\partial w}{\partial y}=0.$

7. $\dfrac{\partial w}{\partial x}+f(ax+by+c)\dfrac{\partial w}{\partial y}=0.$

8. $\dfrac{\partial w}{\partial x}+f\left(\dfrac{y}{x}\right)\dfrac{\partial w}{\partial y}=0.$

9. $x\dfrac{\partial w}{\partial x}+yf(x^ny^m)\dfrac{\partial w}{\partial y}=0.$

10. $\dfrac{\partial w}{\partial x}+yf(e^{\alpha x}y^m)\dfrac{\partial w}{\partial y}=0.$

11. $x\dfrac{\partial w}{\partial x}+f(x^ne^{\alpha y})\dfrac{\partial w}{\partial y}=0.$

1.2. Equations of the Form $f(x,y)\frac{\partial w}{\partial x}+g(x,y)\frac{\partial w}{\partial y}=h(x,y)$

1. $a\dfrac{\partial w}{\partial x}+b\dfrac{\partial w}{\partial y}=f(x).$

2. $\dfrac{\partial w}{\partial x}+a\dfrac{\partial w}{\partial y}=f(x)y^k.$

3. $\dfrac{\partial w}{\partial x}+a\dfrac{\partial w}{\partial y}=f(x)e^{\lambda y}.$

4. $a\dfrac{\partial w}{\partial x}+b\dfrac{\partial w}{\partial y}=f(x)+g(y).$

5. $\dfrac{\partial w}{\partial x}+a\dfrac{\partial w}{\partial y}=f(x)g(y).$

6. $\dfrac{\partial w}{\partial x}+a\dfrac{\partial w}{\partial y}=f(x,y).$

7. $\dfrac{\partial w}{\partial x}+[ay+f(x)]\dfrac{\partial w}{\partial y}=g(x).$

8. $\dfrac{\partial w}{\partial x}+\big[ay+f(x)\big]\dfrac{\partial w}{\partial y}=g(x)h(y).$

9. $\dfrac{\partial w}{\partial x}+\big[f(x)y+g(x)y^k\big]\dfrac{\partial w}{\partial y}=h(x).$

10. $\dfrac{\partial w}{\partial x}+\big[f(x)+g(x)e^{\lambda y}\big]\dfrac{\partial w}{\partial y}=h(x).$

11. $ax\dfrac{\partial w}{\partial x}+by\dfrac{\partial w}{\partial y}=f(x,y).$

12. $f(x)\dfrac{\partial w}{\partial x}+g(y)\dfrac{\partial w}{\partial y}=h_1(x)+h_2(y).$

13. $f(x)\dfrac{\partial w}{\partial x}+g(y)\dfrac{\partial w}{\partial y}=h(x,y).$

14. $f(y)\dfrac{\partial w}{\partial x}+g(x)\dfrac{\partial w}{\partial y}=h(x,y).$

1.3. Equations of the Form $f(x,y)\dfrac{\partial w}{\partial x}+g(x,y)\dfrac{\partial w}{\partial y}=h(x,y)w+r(x,y)$

1. $a\dfrac{\partial w}{\partial x}+b\dfrac{\partial w}{\partial y}=f(x)w.$

2. $a\dfrac{\partial w}{\partial x}+b\dfrac{\partial w}{\partial y}=f(x)w+g(x).$

3. $a\dfrac{\partial w}{\partial x}+b\dfrac{\partial w}{\partial y}=\big[f(x)+g(y)\big]w.$

4. $\dfrac{\partial w}{\partial x}+a\dfrac{\partial w}{\partial y}=f(x,y)w.$

5. $\dfrac{\partial w}{\partial x}+a\dfrac{\partial w}{\partial y}=f(x,y)w+g(x,y).$

6. $ax\dfrac{\partial w}{\partial x}+by\dfrac{\partial w}{\partial y}=f(x)w+g(x).$

7. $ax\dfrac{\partial w}{\partial x}+by\dfrac{\partial w}{\partial y}=f(x,y)w.$

8. $x\dfrac{\partial w}{\partial x}+ay\dfrac{\partial w}{\partial y}=f(x,y)w+g(x,y).$

9. $f(x)\dfrac{\partial w}{\partial x}+g(y)\dfrac{\partial w}{\partial y}=\big[h_1(x)+h_2(y)\big]w.$

10. $f_1(x)\dfrac{\partial w}{\partial x}+f_2(y)\dfrac{\partial w}{\partial y}=aw+g_1(x)+g_2(y).$

11. $f(x)\dfrac{\partial w}{\partial x}+g(y)\dfrac{\partial w}{\partial y}=h(x,y)w+r(x,y).$

12. $f(y)\dfrac{\partial w}{\partial x}+g(x)\dfrac{\partial w}{\partial y}=h(x,y)w+r(x,y).$