



1. First-Order Linear Partial Differential Equations

1.1. Equations of the Form $f(x, y) \frac{\partial w}{\partial x} + g(x, y) \frac{\partial w}{\partial y} = 0$

1. $\frac{\partial w}{\partial x} + [f(x)y + g(x)] \frac{\partial w}{\partial y} = 0.$

2. $\frac{\partial w}{\partial x} + [f(x)y + g(x)y^k] \frac{\partial w}{\partial y} = 0.$

3. $\frac{\partial w}{\partial x} + [f(x)e^{\lambda y} + g(x)] \frac{\partial w}{\partial y} = 0.$

4. $f(x) \frac{\partial w}{\partial x} + g(y) \frac{\partial w}{\partial y} = 0.$

5. $[f(y) + amx^n y^{m-1}] \frac{\partial w}{\partial x} - [g(x) + anx^{n-1} y^m] \frac{\partial w}{\partial y} = 0.$

6. $[e^{\alpha x} f(y) + c\beta] \frac{\partial w}{\partial x} - [e^{\beta y} g(x) + c\alpha] \frac{\partial w}{\partial y} = 0.$

7. $\frac{\partial w}{\partial x} + f(ax + by + c) \frac{\partial w}{\partial y} = 0.$

8. $\frac{\partial w}{\partial x} + f\left(\frac{y}{x}\right) \frac{\partial w}{\partial y} = 0.$

9. $x \frac{\partial w}{\partial x} + y f(x^n y^m) \frac{\partial w}{\partial y} = 0.$

10. $\frac{\partial w}{\partial x} + y f(e^{\alpha x} y^m) \frac{\partial w}{\partial y} = 0.$

11. $x \frac{\partial w}{\partial x} + f(x^n e^{\alpha y}) \frac{\partial w}{\partial y} = 0.$

1.2. Equations of the Form $f(x, y) \frac{\partial w}{\partial x} + g(x, y) \frac{\partial w}{\partial y} = h(x, y)$

1. $a \frac{\partial w}{\partial x} + b \frac{\partial w}{\partial y} = f(x).$

2. $\frac{\partial w}{\partial x} + a \frac{\partial w}{\partial y} = f(x)y^k.$

3. $\frac{\partial w}{\partial x} + a \frac{\partial w}{\partial y} = f(x)e^{\lambda y}.$

$$4. \quad a \frac{\partial w}{\partial x} + b \frac{\partial w}{\partial y} = f(x) + g(y).$$

$$5. \quad \frac{\partial w}{\partial x} + a \frac{\partial w}{\partial y} = f(x)g(y).$$

$$6. \quad \frac{\partial w}{\partial x} + a \frac{\partial w}{\partial y} = f(x, y).$$

$$7. \quad \frac{\partial w}{\partial x} + [ay + f(x)] \frac{\partial w}{\partial y} = g(x).$$

$$8. \quad \frac{\partial w}{\partial x} + [ay + f(x)] \frac{\partial w}{\partial y} = g(x)h(y).$$

$$9. \quad \frac{\partial w}{\partial x} + [f(x)y + g(x)y^k] \frac{\partial w}{\partial y} = h(x).$$

$$10. \quad \frac{\partial w}{\partial x} + [f(x) + g(x)e^{\lambda y}] \frac{\partial w}{\partial y} = h(x).$$

$$11. \quad ax \frac{\partial w}{\partial x} + by \frac{\partial w}{\partial y} = f(x, y).$$

$$12. \quad f(x) \frac{\partial w}{\partial x} + g(y) \frac{\partial w}{\partial y} = h_1(x) + h_2(y).$$

$$13. \quad f(x) \frac{\partial w}{\partial x} + g(y) \frac{\partial w}{\partial y} = h(x, y).$$

$$14. \quad f(y) \frac{\partial w}{\partial x} + g(x) \frac{\partial w}{\partial y} = h(x, y).$$

1.3. Equations of the Form $f(x, y) \frac{\partial w}{\partial x} + g(x, y) \frac{\partial w}{\partial y} = h(x, y)w + r(x, y)$

$$1. \quad a \frac{\partial w}{\partial x} + b \frac{\partial w}{\partial y} = f(x)w.$$

$$2. \quad a \frac{\partial w}{\partial x} + b \frac{\partial w}{\partial y} = f(x)w + g(x).$$

$$3. \quad a \frac{\partial w}{\partial x} + b \frac{\partial w}{\partial y} = [f(x) + g(y)]w.$$

$$4. \quad \frac{\partial w}{\partial x} + a \frac{\partial w}{\partial y} = f(x, y)w.$$

$$5. \quad \frac{\partial w}{\partial x} + a \frac{\partial w}{\partial y} = f(x, y)w + g(x, y).$$

$$6. \quad ax \frac{\partial w}{\partial x} + by \frac{\partial w}{\partial y} = f(x)w + g(x).$$

$$7. \quad ax \frac{\partial w}{\partial x} + by \frac{\partial w}{\partial y} = f(x, y)w.$$

$$8. \quad x \frac{\partial w}{\partial x} + ay \frac{\partial w}{\partial y} = f(x, y)w + g(x, y).$$

$$9. \quad f(x) \frac{\partial w}{\partial x} + g(y) \frac{\partial w}{\partial y} = [h_1(x) + h_2(y)]w.$$

$$10. \quad f_1(x) \frac{\partial w}{\partial x} + f_2(y) \frac{\partial w}{\partial y} = aw + g_1(x) + g_2(y).$$

$$11. \quad f(x) \frac{\partial w}{\partial x} + g(y) \frac{\partial w}{\partial y} = h(x, y)w + r(x, y).$$

$$12. \quad f(y) \frac{\partial w}{\partial x} + g(x) \frac{\partial w}{\partial y} = h(x, y)w + r(x, y).$$

The EqWorld website presents extensive information on solutions to various classes of ordinary differential equations, partial differential equations, integral equations, functional equations, and other mathematical equations.

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<http://eqworld.ipmnet.ru/en/solutions/fpde/fpdetoc1.pdf>