2. First-Order Quasilinear Partial Differential Equations

2.1. Equations of the Form \( f(x, y) \frac{\partial w}{\partial x} + g(x, y) \frac{\partial w}{\partial y} = h(x, y, w) \)

1. \( \frac{\partial w}{\partial x} + a \frac{\partial w}{\partial y} = f(x)w + g(x)w^k. \)

2. \( \frac{\partial w}{\partial x} + a \frac{\partial w}{\partial y} = f(x) + g(x)e^{\lambda w}. \)

3. \( a \frac{\partial w}{\partial x} + b \frac{\partial w}{\partial y} = f(w). \)

4. \( a \frac{\partial w}{\partial x} + b \frac{\partial w}{\partial y} = f(x)g(w). \)

5. \( \frac{\partial w}{\partial x} + a \frac{\partial w}{\partial y} = f(x)g(y)h(w). \)

6. \( ax \frac{\partial w}{\partial x} + by \frac{\partial w}{\partial y} = f(w). \)

7. \( ay \frac{\partial w}{\partial x} + bx \frac{\partial w}{\partial y} = f(w). \)

8. \( ax^n \frac{\partial w}{\partial x} + by^k \frac{\partial w}{\partial y} = f(w). \)

9. \( ay^n \frac{\partial w}{\partial x} + bx^k \frac{\partial w}{\partial y} = f(w). \)

10. \( ae^{\lambda x} \frac{\partial w}{\partial x} + be^{\beta y} \frac{\partial w}{\partial y} = f(w). \)

11. \( ae^{\lambda y} \frac{\partial w}{\partial x} + be^{\beta x} \frac{\partial w}{\partial y} = f(w). \)

12. \( f(x) \frac{\partial w}{\partial x} + g(y) \frac{\partial w}{\partial y} = h(w). \)

13. \( f(y) \frac{\partial w}{\partial x} + g(x) \frac{\partial w}{\partial y} = h(w). \)

2.2. Equations of the Form \( \frac{\partial w}{\partial x} + f(x, y, w) \frac{\partial w}{\partial y} = 0 \)

1. \( \frac{\partial w}{\partial x} + [aw + yf(x)] \frac{\partial w}{\partial y} = 0. \)
### 2. First-Order Quasilinear Partial Differential Equations

2. \[ \frac{\partial w}{\partial x} + \left(aw + f(y)\right) \frac{\partial w}{\partial y} = 0. \]

3. \[ \frac{\partial w}{\partial x} + f(w) \frac{\partial w}{\partial y} = 0. \]

4. \[ \frac{\partial w}{\partial x} + \left[f(w) + ax \right] \frac{\partial w}{\partial y} = 0. \]

5. \[ \frac{\partial w}{\partial x} + \left[f(w) + ay \right] \frac{\partial w}{\partial y} = 0. \]

6. \[ \frac{\partial w}{\partial x} + \left[f(w) + g(x) \right] \frac{\partial w}{\partial y} = 0. \]

7. \[ \frac{\partial w}{\partial x} + \left[f(w) + g(y) \right] \frac{\partial w}{\partial y} = 0. \]

8. \[ \frac{\partial w}{\partial x} + \left[yf(w) + g(x) \right] \frac{\partial w}{\partial y} = 0. \]

9. \[ \frac{\partial w}{\partial x} + \left[xf(w) + yg(w) + h(w) \right] \frac{\partial w}{\partial y} = 0. \]

10. \[ \frac{\partial w}{\partial x} + f(x)g(y)h(w) \frac{\partial w}{\partial y} = 0. \]

### 2.3. Equations of the Form \( \frac{\partial w}{\partial x} + f(x, y, w) \frac{\partial w}{\partial y} = g(x, y, w) \)

1. \[ \frac{\partial w}{\partial x} + aw \frac{\partial w}{\partial y} = f(x). \]

2. \[ \frac{\partial w}{\partial x} + aw \frac{\partial w}{\partial y} = f(y). \]

3. \[ \frac{\partial w}{\partial x} + \left[aw + f(x) \right] \frac{\partial w}{\partial y} = g(x). \]

4. \[ \frac{\partial w}{\partial x} + f(w) \frac{\partial w}{\partial y} = g(x). \]

5. \[ \frac{\partial w}{\partial x} + f(w) \frac{\partial w}{\partial y} = g(y). \]

6. \[ \frac{\partial w}{\partial x} + f(w) \frac{\partial w}{\partial y} = g(w). \]

7. \[ \frac{\partial w}{\partial x} + \left[f(w) + g(x) \right] \frac{\partial w}{\partial y} = h(x). \]

8. \[ \frac{\partial w}{\partial x} + \left[f(w) + g(x) \right] \frac{\partial w}{\partial y} = h(w). \]
9. \( \frac{\partial w}{\partial x} + \left[ f(w) + yg(x) \right] \frac{\partial w}{\partial y} = h(x). \)

10. \( \frac{\partial w}{\partial x} + f(x, w) \frac{\partial w}{\partial y} = g(x). \)

11. \( \frac{\partial w}{\partial x} + f(x, w) \frac{\partial w}{\partial y} = g(w). \)