



## 1. Volterra Integral Equations of the First Kind

### 1-1. Integral equations whose kernels contain power-law functions

1.  $\int_a^x (x-t)y(t) dt = f(x).$
2.  $\int_a^x (Ax + Bt + C)y(t) dt = f(x).$
3.  $\int_a^x (x-t)^n y(t) dt = f(x).$
4.  $\int_a^x \sqrt{x-t} y(t) dt = f(x).$
5.  $\int_a^x \frac{y(t) dt}{\sqrt{x-t}} = f(x).$  *Abel equation.*
6.  $\int_a^x (x-t)^\lambda y(t) dt = f(x).$
7.  $\int_a^x \frac{y(t) dt}{(x-t)^\lambda} = f(x).$  *Generalized Abel equation.*

### 1-2. Integral equations whose kernels contain exponential functions

8.  $\int_a^x e^{\lambda(x-t)} y(t) dt = f(x).$
9.  $\int_a^x e^{\lambda x + \beta t} y(t) dt = f(x).$
10.  $\int_a^x [e^{\lambda(x-t)} - 1] y(t) dt = f(x).$
11.  $\int_a^x [e^{\lambda(x-t)} + b] y(t) dt = f(x).$
12.  $\int_a^x [e^{\lambda(x-t)} - e^{\mu(x-t)}] y(t) dt = f(x).$
13.  $\int_a^x \frac{y(t) dt}{\sqrt{e^{\lambda x} - e^{\lambda t}}} = f(x).$

**1-3. Integral equations whose kernels contain hyperbolic functions**

14. 
$$\int_a^x \cosh[\lambda(x-t)]y(t) dt = f(x).$$

15. 
$$\int_a^x \{\cosh[\lambda(x-t)] - 1\}y(t) dt = f(x).$$

16. 
$$\int_a^x \{\cosh[\lambda(x-t)] + b\}y(t) dt = f(x).$$

17. 
$$\int_a^x \cosh^2[\lambda(x-t)]y(t) dt = f(x).$$

18. 
$$\int_a^x \sinh[\lambda(x-t)]y(t) dt = f(x).$$

19. 
$$\int_a^x \{\sinh[\lambda(x-t)] + b\}y(t) dt = f(x).$$

20. 
$$\int_a^x \sinh(\lambda\sqrt{x-t})y(t) dt = f(x).$$

**1-4. Integral equations whose kernels contain logarithmic functions**

21. 
$$\int_0^x \ln(x-t)y(t) dt = f(x).$$

22. 
$$\int_a^x [\ln(x-t) + A]y(t) dt = f(x).$$

23. 
$$\int_a^x (x-t)[\ln(x-t) + A]y(t) dt = f(x).$$

**1-5. Integral equations whose kernels contain trigonometric functions**

24. 
$$\int_a^x \cos[\lambda(x-t)]y(t) dt = f(x).$$

25. 
$$\int_a^x \{\cos[\lambda(x-t)] - 1\}y(t) dt = f(x).$$

26. 
$$\int_a^x \{\cos[\lambda(x-t)] + b\}y(t) dt = f(x).$$

27. 
$$\int_a^x \sin[\lambda(x-t)]y(t) dt = f(x).$$

28. 
$$\int_a^x \sin(\lambda\sqrt{x-t})y(t) dt = f(x).$$

**1-6. Integral equations whose kernels contain special functions**

29. 
$$\int_a^x J_0(\lambda(x-t))y(t) dt = f(x).$$

30. 
$$\int_a^x J_0(\lambda\sqrt{x-t})y(t) dt = f(x).$$

31. 
$$\int_a^x I_0(\lambda(x-t))y(t) dt = f(x).$$

32. 
$$\int_a^x I_0(\lambda\sqrt{x-t})y(t) dt = f(x).$$

**1-7. Integral equations whose kernels contain arbitrary functions**

33. 
$$\int_a^x [g(x) - g(t)]y(t) dt = f(x).$$

34. 
$$\int_a^x [g(x) - g(t) + b]y(t) dt = f(x).$$

35. 
$$\int_a^x [g(x) + h(t)]y(t) dt = f(x).$$

36. 
$$\int_a^x K(x-t)y(t) dt = f(x).$$

37. 
$$\int_a^x \sqrt{g(x) - g(t)} y(t) dt = f(x).$$

38. 
$$\int_a^x \frac{y(t) dt}{\sqrt{g(x) - g(t)}} = f(x).$$

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The EqWorld website presents extensive information on solutions to various classes of ordinary differential equations, partial differential equations, integral equations, functional equations, and other mathematical equations.

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