



Exact Solutions > Integral Equations > Linear Volterra Integral Equations of the First Kind and Related Integral Equations with Variable Limit of Integration > Generalized Abel Equation

$$7. \int_a^x \frac{y(t) dt}{(x-t)^\lambda} = f(x), \quad 0 < \lambda < 1.$$

Generalized Abel equation (generalized Abel integral equation).

Solution:

$$y(x) = \frac{\sin(\pi\lambda)}{\pi} \frac{d}{dx} \int_a^x \frac{f(t) dt}{(x-t)^{1-\lambda}} = \frac{\sin(\pi\lambda)}{\pi} \left[\frac{f(a)}{(x-a)^{1-\lambda}} + \int_a^x \frac{f'_t(t) dt}{(x-t)^{1-\lambda}} \right].$$

References

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Generalized Abel Equation (Abel Integral Equation)

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