



19. $\int_a^x \{\sinh[\lambda(x-t)] + b\}y(t) dt = f(x), \quad f(a) = 0.$

For $b = 0$, see equation 1.18.

Solution:

$$y(x) = \frac{1}{b} f'_x(x) + \int_a^x R(x-t) f'_t(t) dt,$$
$$R(x) = \frac{\lambda}{b^2} \exp\left(-\frac{\lambda x}{2b}\right) \left[\frac{\lambda}{2bk} \sinh(kx) - \cosh(kx) \right], \quad k = \frac{\lambda\sqrt{1+4b^2}}{2b}.$$

Reference

Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations*, CRC Press, Boca Raton, 1998.