



21. $\int_0^x \ln(x-t)y(t) dt = f(x).$

Solution:

$$y(x) = - \int_0^x f''_{tt}(t) dt \int_0^\infty \frac{(x-t)^z e^{-Cz}}{\Gamma(z+1)} dz - f'_x(0) \int_0^\infty \frac{x^z e^{-Cz}}{\Gamma(z+1)} dz,$$

where $C = \lim_{k \rightarrow \infty} \left(1 + \frac{1}{2} + \dots + \frac{1}{k+1} - \ln k\right) = 0.5772 \dots$ is the Euler constant and $\Gamma(z)$ is the gamma function.

References

Krasnov, M. L., Kiselev, A. I., and Makarenko, G. I., *Problems and Exercises in Integral Equations*, Mir Publ., Moscow, 1971.

Butkovskii, A. G., *Characteristics of Systems With Distributed Parameters* [in Russian], Nauka, Moscow, 1979.

Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations*, CRC Press, Boca Raton, 1998.