22. $\displaystyle \int_a^x [\ln(x-t)+A]y(t)\,dt = f(x).$

Solution:

$$y(x) = -\frac{d}{dx}\int_a^x \nu_A(x-t)f(t)\,dt, \qquad \nu_A(x) = \frac{d}{dx}\int_0^\infty \frac{x^z e^{(A-\mathcal{C})z}}{\Gamma(z+1)}\,dz,$$

where $\mathcal{C} = 0.5772\ldots$ is the Euler constant and $\Gamma(z)$ is the gamma function.

For $a = 0$, the solution can be written in the form

$$y(x) = -\int_0^x f''_{tt}(t)\,dt \int_0^\infty \frac{(x-t)^z e^{(A-\mathcal{C})z}}{\Gamma(z+1)}\,dz - f'_x(0)\int_0^\infty \frac{x^z e^{(A-\mathcal{C})z}}{\Gamma(z+1)}\,dz.$$

References

Samko, S. G., Kilbas, A. A., and Marichev, O. I., *Fractional Integrals and Derivatives. Theory and Applications*, Gordon & Breach Sci. Publ., New York, 1993.

Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations*, CRC Press, Boca Raton, 1998.

