



23. $\int_a^x (x-t)[\ln(x-t) + A]y(t) dt = f(x), \quad f(a) = 0.$

Solution:

$$y(x) = -\frac{d^2}{dx^2} \int_a^x \nu_A(x-t)f(t) dt, \quad \nu_A(x) = \frac{d}{dx} \int_0^\infty \frac{x^z e^{(A-C)z}}{\Gamma(z+1)} dz,$$

where $C = 0.5772\dots$ is the Euler constant and $\Gamma(z)$ is the gamma function.

References

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