36. $\displaystyle\int_a^x K(x-t)y(t)\,dt = f(x).$

1°. Let $K(0)=1$ and $f(a)=0$. Differentiating the equation with respect to x yields a Volterra equation of the second kind:

$$y(x)+\int_a^x K'_x(x-t)y(t)\,dt = f'_x(x).$$

The solution of this equation can be represented in the form

$$y(x)=f'_x(x)+\int_a^x R(x-t)f'_t(t)\,dt. \qquad (1)$$

Here, the resolvent $R(x)$ is related to the kernel $K(x)$ of the original equation by

$$R(x)=\mathfrak{L}^{-1}\left[\frac{1}{p\widetilde{K}(p)}-1\right], \qquad \widetilde{K}(p)=\mathfrak{L}\big[K(x)\big],$$

where $\mathfrak{L}$ and $\mathfrak{L}^{-1}$ are the operators of the direct and inverse Laplace transforms, respectively.

$$\widetilde{K}(p)=\mathfrak{L}\big[K(x)\big]=\int_0^\infty e^{-px}K(x)\,dx, \qquad R(x)=\mathfrak{L}^{-1}\big[\widetilde{R}(p)\big]=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} e^{px}\widetilde{R}(p)\,dp.$$

2°. Let $K(x)$ have an integrable power-law singularity at $x=0$. Denote by $w=w(x)$ the solution of the simpler auxiliary equation (compared with the original equation) with $a=0$ and constant right-hand side $f\equiv 1$,

$$\int_0^x K(x-t)w(t)\,dt = 1. \qquad (2)$$

Then the solution of the original integral equation with arbitrary right-hand side is expressed in terms of w as follows:

$$y(x)=\frac{d}{dx}\int_a^x w(x-t)f(t)\,dt = f(a)w(x-a)+\int_a^x w(x-t)f'_t(t)\,dt.$$

Reference

Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations*, CRC Press, Boca Raton, 1998.

