37. $\displaystyle\int_a^x \sqrt{g(x)-g(t)}\,y(t)\,dt = f(x), \qquad f(a) = 0, \quad g'_x(x) > 0.$

Solution:

$$y(x) = \frac{2}{\pi} g'_x(x) \left(\frac{1}{g'_x(x)} \frac{d}{dx} \right)^2 \int_a^x \frac{f(t) g'_t(t)\,dt}{\sqrt{g(x)-g(t)}}.$$

Reference

Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations*, CRC Press, Boca Raton, 1998.

