



5.  $y(x) + A \int_a^x (x-t)^n y(t) dt = f(x), \quad n = 1, 2, \dots$

1°. Differentiating the equation  $n+1$  times with respect to  $x$  yields an  $(n+1)$ st-order linear ordinary differential equation with constant coefficients for  $y = y(x)$ :

$$y_x^{(n+1)} + An! y = f_x^{(n+1)}(x).$$

This equation under the initial conditions  $y(a) = f(a), y'_x(a) = f'_x(a), \dots, y_x^{(n)}(a) = f_x^{(n)}(a)$  determines the solution of the original integral equation.

2°. Solution:

$$y(x) = f(x) + \int_a^x R(x-t)f(t) dt,$$
$$R(x) = \frac{1}{n+1} \sum_{k=0}^n \exp(\sigma_k x) [\sigma_k \cos(\beta_k x) - \beta_k \sin(\beta_k x)],$$

where the coefficients  $\sigma_k$  and  $\beta_k$  are given by

$$\sigma_k = |An!|^{\frac{1}{n+1}} \cos\left(\frac{2\pi k}{n+1}\right), \quad \beta_k = |An!|^{\frac{1}{n+1}} \sin\left(\frac{2\pi k}{n+1}\right) \quad \text{for } A < 0,$$
$$\sigma_k = |An!|^{\frac{1}{n+1}} \cos\left(\frac{2\pi k + \pi}{n+1}\right), \quad \beta_k = |An!|^{\frac{1}{n+1}} \sin\left(\frac{2\pi k + \pi}{n+1}\right) \quad \text{for } A > 0.$$

### Reference

**Polyanin, A. D. and Manzhirov, A. V.,** *Handbook of Integral Equations*, CRC Press, Boca Raton, 1998.