5. $y(x) + A\int_a^x (x-t)^n y(t)\,dt = f(x), \qquad n = 1, 2, \dots$

1°. Differentiating the equation $n+1$ times with respect to x yields an $(n+1)$st-order linear ordinary differential equation with constant coefficients for $y = y(x)$:

$$y_x^{(n+1)} + An!\,y = f_x^{(n+1)}(x).$$

This equation under the initial conditions $y(a) = f(a)$, $y'_x(a) = f'_x(a)$, $\dots$, $y_x^{(n)}(a) = f_x^{(n)}(a)$ determines the solution of the original integral equation.

2°. Solution:

$$y(x) = f(x) + \int_a^x R(x-t) f(t)\,dt,$$

$$R(x) = \frac{1}{n+1}\sum_{k=0}^{n} \exp(\sigma_k x)\big[\sigma_k \cos(\beta_k x) - \beta_k \sin(\beta_k x)\big],$$

where the coefficients σ_k and β_k are given by

$$\sigma_k = |An!|^{\frac{1}{n+1}} \cos\left(\frac{2\pi k}{n+1}\right), \qquad \beta_k = |An!|^{\frac{1}{n+1}} \sin\left(\frac{2\pi k}{n+1}\right) \qquad \text{for} \quad A < 0,$$

$$\sigma_k = |An!|^{\frac{1}{n+1}} \cos\left(\frac{2\pi k + \pi}{n+1}\right), \qquad \beta_k = |An!|^{\frac{1}{n+1}} \sin\left(\frac{2\pi k + \pi}{n+1}\right) \qquad \text{for} \quad A > 0.$$

Reference

Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations*, CRC Press, Boca Raton, 1998.

