13. $\displaystyle y(x) - \lambda \int_0^x J_0(x-t)y(t)\,dt = f(x).$

Here, $J_0(z)$ is the Bessel function of the first kind.

Solution:

$$y(x) = f(x) + \int_0^x R(x-t)f(t)\,dt,$$

where

$$R(x) = \lambda\cos\bigl(\sqrt{1-\lambda^2}\,x\bigr) + \frac{\lambda^2}{\sqrt{1-\lambda^2}}\sin\bigl(\sqrt{1-\lambda^2}\,x\bigr) + \frac{\lambda}{\sqrt{1-\lambda^2}}\int_0^x \sin\bigl[\sqrt{1-\lambda^2}\,(x-t)\bigr]\frac{J_1(t)}{t}\,dt.$$

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