16. $y(x) + \int_a^x (x-t)g(t)y(t)\,dt = f(x).$

1°. Solution:

$$y(x) = f(x) + \frac{1}{W}\int_a^x \bigl[Y_1(x)Y_2(t) - Y_2(x)Y_1(t)\bigr]\,g(t)f(t)\,dt, \qquad (1)$$

where $Y_1 = Y_1(x)$ and $Y_2 = Y_2(x)$ are two linearly independent solutions ($Y_1/Y_2 \not\equiv \text{const}$) of the second-order linear homogeneous differential equation $Y''_{xx} + g(x)Y = 0$. In this case, the Wronskian is a constant: $W = Y_1(Y_2)'_x - Y_2(Y_1)'_x \equiv \text{const}$.

2°. Given only one nontrivial solution $Y_1 = Y_1(x)$ of the linear homogeneous differential equation $Y''_{xx} + g(x)Y = 0$, one can obtain the solution of the integral equation by formula (1) with

$$W = 1, \qquad Y_2(x) = Y_1(x)\int_b^x \frac{d\xi}{Y_1^2(\xi)},$$

where b is an arbitrary number.

Reference

Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations*, CRC Press, Boca Raton, 1998.

