17. \( y(x) + \int_{a}^{x} K(x-t)y(t)\,dt = f(x). \)

*Renewal equation.*

1°. To solve this integral equation, direct and inverse Laplace transforms are used. The solution can be represented in the form

\[
y(x) = f(x) - \int_{a}^{x} R(x-t)f(t)\,dt.
\]

Here, the resolvent \( R(x) \) is expressed via the kernel \( K(x) \) of the original equation as follows:

\[
R(x) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \tilde{R}(p)e^{px}\,dp, \quad \tilde{R}(p) = \frac{\tilde{K}(p)}{1 + \tilde{K}(p)}, \quad \tilde{K}(p) = \int_{0}^{\infty} K(x)e^{-px}\,dx.
\]

2°. Let \( w = w(x) \) be the solution of the simpler auxiliary equation with \( a = 0 \) and \( f \equiv 1 \):

\[
w(x) + \int_{0}^{x} K(x-t)w(t)\,dt = 1. \tag{2}
\]

Then the solution of the original integral equation with arbitrary \( f = f(x) \) is expressed via the solution of the auxiliary equation (2) as

\[
y(x) = \frac{d}{dx} \int_{a}^{x} w(x-t)f(t)\,dt = f(a)w(x-a) + \int_{a}^{x} w(x-t)f'(t)\,dt.
\]

**References**


