3. $\displaystyle\int_a^b \frac{y(t)}{|x-t|^k}\,dt = f(x), \qquad 0 < k < 1.$

It is assumed that $|a| + |b| < \infty$. Solution:

$$y(x) = \frac{1}{2\pi}\cot\left(\tfrac{1}{2}\pi k\right)\frac{d}{dx}\int_a^x \frac{f(t)\,dt}{(x-t)^{1-k}} - \frac{1}{\pi^2}\cos^2\left(\tfrac{1}{2}\pi k\right)\int_a^x \frac{Z(t)F(t)}{(x-t)^{1-k}}\,dt,$$

where

$$Z(t) = (t-a)^{\frac{1+k}{2}}(b-t)^{\frac{1-k}{2}}, \qquad F(t) = \frac{d}{dt}\left[\int_a^t \frac{d\tau}{(t-\tau)^k}\int_\tau^b \frac{f(s)\,ds}{Z(s)(s-\tau)^{1-k}}\right].$$

References

Gakhov, F. D., *Boundary Value Problems* [in Russian], Nauka, Moscow, 1977.

Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations*, CRC Press, Boca Raton, 1998.

