10. $\displaystyle\int_a^b \frac{y(t)\,dt}{t-x} = f(x).$

This equation is encountered in hydrodynamics in solving the problem on the flow of an ideal inviscid fluid around a thin profile ($a \le x \le b$). It is assumed that $|a| + |b| < \infty$.

1°. The solution bounded at the endpoints is

$$y(x) = -\frac{1}{\pi^2}\sqrt{(x-a)(b-x)}\int_a^b \frac{f(t)}{\sqrt{(t-a)(b-t)}}\,\frac{dt}{t-x},$$

provided that

$$\int_a^b \frac{f(t)\,dt}{\sqrt{(t-a)(b-t)}} = 0.$$

2°. The solution bounded at the endpoint $x = a$ and unbounded at the endpoint $x = b$ is

$$y(x) = -\frac{1}{\pi^2}\sqrt{\frac{x-a}{b-x}}\int_a^b \sqrt{\frac{b-t}{t-a}}\,\frac{f(t)}{t-x}\,dt.$$

3°. The solution unbounded at the endpoints is

$$y(x) = -\frac{1}{\pi^2\sqrt{(x-a)(b-x)}}\left[\int_a^b \frac{\sqrt{(t-a)(b-t)}}{t-x}\,f(t)\,dt + C\right],$$

where C is an arbitrary constant. The formula $\displaystyle\int_a^b y(t)\,dt = C/\pi$ holds.

References

Gakhov, F. D., *Boundary Value Problems* [in Russian], Nauka, Moscow, 1977.

Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations*, CRC Press, Boca Raton, 1998.

