12. $\displaystyle\int_a^b \ln|x-t|\,y(t)\,dt = f(x).$

Carleman's equation.

1°. Solution for $b-a \neq 4$:

$$y(x) = \frac{1}{\pi^2\sqrt{(x-a)(b-x)}}\left[\int_a^b \frac{\sqrt{(t-a)(b-t)}\,f'_t(t)\,dt}{t-x} + \frac{1}{\ln\left[\frac{1}{4}(b-a)\right]}\int_a^b \frac{f(t)\,dt}{\sqrt{(t-a)(b-t)}}\right].$$

2°. If $b-a=4$, then for the equation to be solvable, the condition

$$\int_a^b f(t)(t-a)^{-1/2}(b-t)^{-1/2}\,dt = 0$$

must be satisfied. In this case, the solution has the form

$$y(x) = \frac{1}{\pi^2\sqrt{(x-a)(b-x)}}\left[\int_a^b \frac{\sqrt{(t-a)(b-t)}\,f'_t(t)\,dt}{t-x} + C\right],$$

where C is an arbitrary constant.

References

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