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12.
$$\int_a^b \ln|x-t| y(t) dt = f(x).$$

Carleman's equation.

1°. Solution for $b-a \neq 4$:

$$y(x) = \frac{1}{\pi^2 \sqrt{(x-a)(b-x)}} \left[\int_a^b \frac{\sqrt{(t-a)(b-t)} f'_t(t) dt}{t-x} + \frac{1}{\ln\left[\frac{1}{4}(b-a)\right]} \int_a^b \frac{f(t) dt}{\sqrt{(t-a)(b-t)}} \right].$$

2°. If $b-a = 4$, then for the equation to be solvable, the condition

$$\int_a^b f(t)(t-a)^{-1/2}(b-t)^{-1/2} dt = 0$$

must be satisfied. In this case, the solution has the form

$$y(x) = \frac{1}{\pi^2 \sqrt{(x-a)(b-x)}} \left[\int_a^b \frac{\sqrt{(t-a)(b-t)} f'_t(t) dt}{t-x} + C \right],$$

where C is an arbitrary constant.

References

Gakhov, F. D., *Boundary Value Problems* [in Russian], Nauka, Moscow, 1977.

Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations*, CRC Press, Boca Raton, 1998.

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