19. $\displaystyle \int_0^{2\pi} \cot\left(\frac{t-x}{2}\right) y(t)\,dt = f(x), \qquad 0 \le x \le 2\pi.$

Here, the integral is understood in the sense of the Cauchy principal value and the right-hand side is assumed to satisfy the condition $\int_0^{2\pi} f(t)\,dt = 0$.

Solution:

$$y(x) = -\frac{1}{4\pi^2} \int_0^{2\pi} \cot\left(\frac{t-x}{2}\right) f(t)\,dt + C,$$

where C is an arbitrary constant.

It follows from the solution that $\int_0^{2\pi} y(t)\,dt = 2\pi C$.

The equation and its solution form a Hilbert transform pair (in the asymmetric form).

References

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