



22.
$$\int_{-\infty}^{\infty} K(x-t)y(t) dt = f(x).$$

The Fourier transform is used to solve this equation.

1°. Solution:

$$y(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\tilde{f}(u)}{\tilde{K}(u)} e^{iux} du,$$

where

$$\tilde{f}(u) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)e^{-iux} dx, \quad \tilde{K}(u) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} K(x)e^{-iux} dx.$$

The following statement is valid. Let $f(x) \in L_2(-\infty, \infty)$ and $K(x) \in L_1(-\infty, \infty)$. Then for a solution $y(x) \in L_2(-\infty, \infty)$ of the integral equation to exist, it is necessary and sufficient that $\tilde{f}(u)/\tilde{K}(u) \in L_2(-\infty, \infty)$.

2°. Let the function $P(s)$ defined by the formula

$$\frac{1}{P(s)} = \int_{-\infty}^{\infty} e^{-st} K(t) dt$$

be a polynomial of degree n with real roots of the form

$$P(s) = \left(1 - \frac{s}{a_1}\right) \left(1 - \frac{s}{a_2}\right) \dots \left(1 - \frac{s}{a_n}\right).$$

Then the solution of the integral equation is given by

$$y(x) = P(D)f(x), \quad D = \frac{d}{dx}.$$

References

- Hirschman, I. I. and Widder, D. V.**, *The Convolution Transform*, Princeton Univ. Press, Princeton–New Jersey, 1955.
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