3. \( Ay(x) + \frac{B}{\pi} \int_{-1}^{1} \frac{y(t) dt}{t - x} = f(x), \quad -1 < x < 1. \)

In the equation and its solutions, singular integrals are understood in the sense of the Cauchy principal value. Without loss of generality we may assume that \( A^2 + B^2 = 1. \)

1°. The solution bounded at the endpoints:

\[
y(x) = Af(x) - \frac{B}{\pi} \int_{-1}^{1} \frac{g(x)}{g(t)} \frac{f(t) dt}{t - x}, \quad g(x) = (1 + x)^{\alpha}(1 - x)^{1-\alpha},
\]

where \( \alpha \) is the solution of the trigonometric equation

\[
A + B \cot(\pi \alpha) = 0
\]

on the interval \( 0 < \alpha < 1. \) This solution \( y(x) \) exists if and only if \( \int_{-1}^{1} \frac{f(t)}{g(t)} dt = 0. \)

2°. The solution bounded at the endpoint \( x = 1 \) and unbounded at the endpoint \( x = -1: \)

\[
y(x) = Af(x) - \frac{B}{\pi} \int_{-1}^{1} \frac{g(x)}{g(t)} \frac{f(t) dt}{t - x}, \quad g(x) = (1 + x)^{\alpha}(1 - x)^{-\alpha},
\]

where \( \alpha \) is the solution of the trigonometric equation (2) on the interval \( -1 < \alpha < 0. \)

3°. The solution unbounded at the endpoints:

\[
y(x) = Af(x) - \frac{B}{\pi} \int_{-1}^{1} \frac{g(x)}{g(t)} \frac{f(t) dt}{t - x} + Cg(x), \quad g(x) = (1 + x)^{\alpha}(1 - x)^{1-\alpha},
\]

where \( C \) is an arbitrary constant and \( \alpha \) is the solution of the trigonometric equation (2) on the interval \( -1 < \alpha < 0. \)

References
