4. $y(x) - \lambda \int_0^1 \left(\frac{1}{t-x} - \frac{1}{x+t-2xt} \right) y(t)\,dt = f(x), \quad 0 < x < 1.$

Tricomi's equation (Tricomi's integral equation). In the equation and its solutions, singular integrals are understood in the sense of the Cauchy principal value.

Solution:

$$y(x) = \frac{1}{1+\lambda^2\pi^2}\left[f(x) + \int_0^1 \frac{t^\alpha (1-x)^\alpha}{x^\alpha (1-t)^\alpha} \left(\frac{1}{t-x} - \frac{1}{x+t-2xt} \right) f(t)\,dt \right] + \frac{C(1-x)^\beta}{x^{1+\beta}},$$

$$\alpha = \frac{2}{\pi}\arctan(\lambda\pi) \ \ (-1 < \alpha < 1), \quad \tan\frac{\beta\pi}{2} = \lambda\pi \ \ (-2 < \beta < 0),$$

where C is an arbitrary constant.

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