6. $y(x) - \lambda \int_{-\infty}^{\infty} e^{-|x-t|} y(t)\,dt = 0, \qquad \lambda > 0.$

Lalesco–Picard equation.

Solution:

$$y(x) = \begin{cases} C_1 \exp\left(x\sqrt{1-2\lambda}\right) + C_2 \exp\left(-x\sqrt{1-2\lambda}\right) & \text{for } 0 < \lambda < \frac{1}{2}, \\ C_1 + C_2 x & \text{for } \lambda = \frac{1}{2}, \\ C_1 \cos\left(x\sqrt{2\lambda-1}\right) + C_2 \sin\left(x\sqrt{2\lambda-1}\right) & \text{for } \lambda > \frac{1}{2}, \end{cases}$$

where C_1 and C_2 are arbitrary constants.

References

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