13. $\displaystyle Ay(x) - \frac{B}{2\pi}\int_0^{2\pi} \cot\left(\frac{t-x}{2}\right) y(t)\,dt = f(x), \qquad 0 \le x \le 2\pi.$

Here, the integral is understood in the sense of the Cauchy principal value. Without loss of generality we may assume that $A^2 + B^2 = 1$.

Solution:

$$y(x) = Af(x) + \frac{B}{2\pi}\int_0^{2\pi} \cot\left(\frac{t-x}{2}\right) f(t)\,dt + \frac{B^2}{2\pi A}\int_0^{2\pi} f(t)\,dt.$$

References

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