4. $\displaystyle\int_0^x y(t)y(x-t)\,dt = A\sin(\lambda x).$

Solutions:

$$y = \pm\sqrt{A\lambda}\,J_0(\lambda x),$$

where $J_0(z)$ is the Bessel function.

Reference

Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations*, CRC Press, Boca Raton, 1998.

