2. $\displaystyle\int_0^1 f(t)y(t)y(xt)\,dt = A.$

1°. Solutions:

$$y_1(x) = \sqrt{A/I_0}, \qquad y_2(x) = -\sqrt{A/I_0},$$
$$y_3(x) = q(I_1 x - I_2), \qquad y_4(x) = -q(I_1 x - I_2),$$

where

$$I_m = \int_0^1 t^m f(t)\,dt, \quad q = \left(\frac{A}{I_0 I_2^2 - I_1^2 I_2}\right)^{1/2}, \qquad m = 0, 1, 2.$$

The integral equation has some other (more complicated) solutions of the polynomial form $y(x) = \sum_{k=0}^{n} B_k x^k$, where the constants B_k can be found from the corresponding system of algebraic equations.

2°. Solutions:

$$y_5(x) = q(I_1 x^C - I_2), \qquad y_6(x) = -q(I_1 x^C - I_2),$$
$$q = \left(\frac{A}{I_0 I_2^2 - I_1^2 I_2}\right)^{1/2}, \quad I_m = \int_0^1 t^{mC} f(t)\,dt, \qquad m = 0, 1, 2,$$

where C is an arbitrary constant.

The equation has more complicated solutions of the form $y(x) = \sum_{k=0}^{n} B_k x^{kC}$, where C is an arbitrary constant and the coefficients B_k can be found from the corresponding system of algebraic equations.

3°. Solutions:

$$y_7(x) = p(J_0 \ln x - J_1), \qquad y_8(x) = -p(J_0 \ln x - J_1),$$
$$p = \left(\frac{A}{J_0^2 J_2 - J_0 J_1^2}\right)^{1/2}, \quad J_m = \int_0^1 (\ln t)^m f(t)\,dt.$$

The equation has more complicated solutions of the form $y(x) = \sum_{k=0}^{n} E_k (\ln x)^k$, where the constants E_k can be found from the corresponding system of algebraic equations.

Reference

Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations*, CRC Press, Boca Raton, 1998.

