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1.3. Heat Equation of the Form $\frac{\partial w}{\partial t} = a \frac{\partial^2 w}{\partial x^2} + b \frac{\partial w}{\partial x} + cw + \Phi(x, t)$

Convective heat (diffusion) equation with a source.

The substitution

$$w(x, t) = \exp(\beta t + \mu x)u(x, t), \quad \beta = c - \frac{b^2}{4a}, \quad \mu = -\frac{b}{2a}$$

leads to the nonhomogeneous heat equation

$$\frac{\partial u}{\partial t} = a \frac{\partial^2 u}{\partial x^2} + \exp(-\beta t - \mu x)\Phi(x, t),$$

which is considered in [Subsection 1.1](#) and [Subsection 1.2](#).

Reference

Polyanin, A. D., *Handbook of Linear Partial Differential Equations for Engineers and Scientists*, Chapman & Hall/CRC, 2002.

Convective Heat Equation with a Source

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<http://eqworld.ipmnet.ru/en/solutions/lpde/lpde103.pdf>