1.7. **Heat Equation of the Form** \( \frac{\partial w}{\partial t} = a \left( \frac{\partial^2 w}{\partial r^2} + \frac{2}{r} \frac{\partial w}{\partial r} \right) + \Phi(r, t) \)

**Nonhomogeneous heat (diffusion) equation with central symmetry.**

1.7-1. **Solutions of boundary value problems in terms of the Green’s function.**

We consider boundary value problems in domain \( 0 \leq r \leq R \) with the general initial condition \( w = f(r) \) at \( t = 0 \)

and various homogeneous boundary conditions (the solutions bounded at \( r = 0 \) are sought). The solution can be represented in terms of the Green’s function as

\[
 w(x, t) = \int_0^R f(\xi) G(r, \xi, t) d\xi + \int_0^t \int_0^R \Phi(\xi, \tau) G(r, \xi, t - \tau) d\xi d\tau.
\]

1.7-2. **Domain:** \( 0 \leq r \leq R \). **First boundary value problem for heat equation.**

A boundary condition is prescribed:

\[ w = 0 \quad \text{at} \quad r = R. \]

Green’s function:

\[
 G(r, \xi, t) = \frac{2\xi}{Rr} \sum_{n=1}^{\infty} \sin \left( \frac{n\pi r}{R} \right) \sin \left( \frac{n\pi \xi}{R} \right) \exp \left( -\frac{an^2\pi^2 t}{R^2} \right).
\]

1.7-3. **Domain:** \( 0 \leq r \leq R \). **Second boundary value problem for heat equation.**

A boundary condition is prescribed:

\[ \frac{\partial w}{\partial r} = 0 \quad \text{at} \quad r = R. \]

Green’s function:

\[
 G(r, \xi, t) = \frac{3\xi^2}{R^2} + \frac{2\xi}{Rr} \sum_{n=1}^{\infty} \frac{\mu_n + 1}{\mu_n^2} \sin \left( \frac{\mu_n r}{R} \right) \sin \left( \frac{\mu_n \xi}{R} \right) \exp \left( -\frac{a\mu_n^2 t}{R^2} \right),
\]

where the \( \mu_n \) are positive roots of the transcendental equation \( \tan \mu - \mu = 0 \). The first five roots are

\[
 \mu_1 = 4.4934, \quad \mu_2 = 7.7253, \quad \mu_3 = 10.9041, \quad \mu_4 = 14.0662, \quad \mu_5 = 17.2208.
\]

**References**
