4.2. Generalized Tricomi Equation $\dfrac{\partial^2 w}{\partial x^2} + f(x)\dfrac{\partial^2 w}{\partial y^2} = 0$

4.2-1. Particular solutions of the generalized Tricomi equation:

$$w = C_1 xy + C_2 y + C_3 x + C_4,$$
$$w = C_1 y^2 + C_2 xy + C_3 y + C_4 x - 2C_1 \int_a^x (x-t) f(t)\,dt + C_5,$$
$$w = C_1 y^3 + C_2 xy + C_3 y + C_4 x - 6C_1 y \int_a^x (x-t) f(t)\,dt + C_5,$$
$$w = (C_1 x + C_2) y^2 + C_3 xy + C_4 y + C_5 x - 2\int_a^x (x-t)(C_1 t + C_2) f(t)\,dt + C_6,$$

where C_1, C_2, C_3, C_4, C_5, and C_6 are arbitrary constants, a is any number.

4.2-2. Separable particular solution of the generalized Tricomi equation:

$$w = (C_1 e^{\lambda y} + C_2 e^{-\lambda y}) H(x),$$

where C_1, C_2, and λ are arbitrary constants, and the function $H = H(x)$ is determined by the ordinary differential equation $H''_{xx} + \lambda^2 f(x) H = 0$.

4.2-3. Separable particular solution of the generalized Tricomi equation:

$$w = [C_1 \sin(\lambda y) + C_2 \cos(\lambda y)] Z(x),$$

where C_1, C_2, and λ are arbitrary constants, and the function $Z = Z(x)$ is determined by the ordinary differential equation $Z''_{xx} - \lambda^2 f(x) Z = 0$.

4.2-4. Particular solutions of the generalized Tricomi equation with even powers of y:

$$w = \sum_{k=0}^{n} \varphi_k(x) y^{2k},$$

where the functions $\varphi_k = \varphi_k(x)$ are defined by the recurrence relations

$$\varphi_n(x) = A_n x + B_n, \quad \varphi_{k-1}(x) = A_k x + B_k - 2k(2k-1)\int_a^x (x-t) f(t) \varphi_k(t)\,dt,$$

where A_k and B_k are arbitrary constants ($k = n, \ldots, 1$), a is any number.

4.2-5. Particular solutions of the generalized Tricomi equation with odd powers of y:

$$w = \sum_{k=0}^{n} \psi_k(x) y^{2k+1},$$

where the functions $\psi_k = \psi_k(x)$ are defined by the recurrence relations

$$\psi_n(x) = A_n x + B_n, \quad \psi_{k-1}(x) = A_k x + B_k - 2k(2k+1)\int_a^x (x-t) f(t) \psi_k(t)\,dt,$$

where A_k and B_k are arbitrary constants ($k = n, \ldots, 1$), and a is any number.

Reference

Polyanin, A. D., *Handbook of Linear Partial Differential Equations for Engineers and Scientists*, Chapman & Hall/CRC, 2002.

