4.2. Generalized Tricomi Equation $\frac{\partial^2 w}{\partial x^2} + f(x) \frac{\partial^2 w}{\partial y^2} = 0$

4.2-1. Particular solutions of the generalised Tricomi equation:

\[ w = C_1 xy + C_2 y + C_3 x + C_4, \]
\[ w = C_1 y^2 + C_2 xy + C_3 y + C_4 x - 2C_1 \int_a^x (x-t) f(t) \, dt + C_5, \]
\[ w = C_1 y^3 + C_2 xy + C_3 y + C_4 x - 6C_1 y \int_a^x (x-t) f(t) \, dt + C_5, \]
\[ w = (C_1 x + C_2) y^2 + C_3 xy + C_4 y + C_5 x - 2 \int_a^x (x-t)(C_1 t + C_2) f(t) \, dt + C_6, \]

where $C_1, C_2, C_3, C_4, C_5$, and $C_6$ are arbitrary constants, $a$ is any number.

4.2-2. Separable particular solution of the generalised Tricomi equation:

\[ w = (C_1 e^{\lambda y} + C_2 e^{-\lambda y}) H(x), \]

where $C_1, C_2$, and $\lambda$ are arbitrary constants, and the function $H = H(x)$ is determined by the ordinary differential equation $H''_x + \lambda^2 f(x) H = 0$.

4.2-3. Separable particular solution of the generalised Tricomi equation:

\[ w = [C_1 \sin(\lambda y) + C_2 \cos(\lambda y)] Z(x), \]

where $C_1, C_2$, and $\lambda$ are arbitrary constants, and the function $Z = Z(x)$ is determined by the ordinary differential equation $Z''_x - \lambda^2 f(x) Z = 0$.

4.2-4. Particular solutions of the generalised Tricomi equation with even powers of $y$:

\[ w = \sum_{k=0}^{n} \varphi_k(x) y^{2k}, \]

where the functions $\varphi_k = \varphi_k(x)$ are defined by the recurrence relations

\[ \varphi_n(x) = A_n x + B_n, \quad \varphi_{n-1}(x) = A_k x + B_k - 2k(2k - 1) \int_a^x (x-t) \varphi_k(t) \, dt, \]

where $A_k$ and $B_k$ are arbitrary constants ($k = n, \ldots, 1$), $a$ is any number.

4.2-5. Particular solutions of the generalised Tricomi equation with odd powers of $y$:

\[ w = \sum_{k=0}^{n} \psi_k(x) y^{2k+1}, \]

where the functions $\psi_k = \psi_k(x)$ are defined by the recurrence relations

\[ \psi_n(x) = A_n x + B_n, \quad \psi_{n-1}(x) = A_k x + B_k - 2k(2k + 1) \int_a^x (x-t) \psi_k(t) \, dt, \]

where $A_k$ and $B_k$ are arbitrary constants ($k = n, \ldots, 1$), and $a$ is any number.

Reference


Generalized Tricomi Equation