3. Nonlinear Elliptic Equations

3.1. Nonlinear Heat Equations of the Form $\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}=f(w)$

1. $\dfrac{\partial^2 w}{\partial x^2}+\dfrac{\partial^2 w}{\partial y^2}=aw+bw^n.$

2. $\dfrac{\partial^2 w}{\partial x^2}+\dfrac{\partial^2 w}{\partial y^2}=aw^n+bw^{2n-1}.$

3. $\dfrac{\partial^2 w}{\partial x^2}+\dfrac{\partial^2 w}{\partial y^2}=ae^{\beta w}.$

4. $\dfrac{\partial^2 w}{\partial x^2}+\dfrac{\partial^2 w}{\partial y^2}=ae^{\beta w}+be^{2\beta w}.$

5. $\dfrac{\partial^2 w}{\partial x^2}+\dfrac{\partial^2 w}{\partial y^2}=\alpha w\ln(\beta w).$

6. $\dfrac{\partial^2 w}{\partial x^2}+\dfrac{\partial^2 w}{\partial y^2}=\alpha\sin(\beta w).$

7. $\dfrac{\partial^2 w}{\partial x^2}+\dfrac{\partial^2 w}{\partial y^2}=f(w).$

3.2. Heat Equations of the Form $\frac{\partial}{\partial x}\left[f(x)\frac{\partial w}{\partial x}\right]+\frac{\partial}{\partial y}\left[g(y)\frac{\partial w}{\partial y}\right]=f(w)$

1. $\dfrac{\partial}{\partial x}\left(ax^n\dfrac{\partial w}{\partial x}\right)+\dfrac{\partial}{\partial y}\left(by^m\dfrac{\partial w}{\partial y}\right)=f(w).$ *Anisotropic heat/diffusion equation.*

2. $a\dfrac{\partial^2 w}{\partial x^2}+\dfrac{\partial}{\partial y}\left(be^{\mu y}\dfrac{\partial w}{\partial y}\right)=f(w).$ *Anisotropic heat/diffusion equation.*

3. $\dfrac{\partial}{\partial x}\left(ae^{\beta x}\dfrac{\partial w}{\partial x}\right)+\dfrac{\partial}{\partial y}\left(be^{\mu y}\dfrac{\partial w}{\partial y}\right)=f(w).$ *Anisotropic heat/diffusion equation.*

4. $\dfrac{\partial}{\partial x}\left[f(x)\dfrac{\partial w}{\partial x}\right]+\dfrac{\partial}{\partial y}\left[g(y)\dfrac{\partial w}{\partial y}\right]=kw\ln w.$ *Anisotropic heat/diffusion equation.*

3.3. Heat Equations of the Form $\frac{\partial}{\partial x}\left[f(w)\frac{\partial w}{\partial x}\right]+\frac{\partial}{\partial y}\left[g(w)\frac{\partial w}{\partial y}\right]=h(w)$

1. $\dfrac{\partial^2 w}{\partial x^2}+\dfrac{\partial}{\partial y}\left[(\alpha w+\beta)\dfrac{\partial w}{\partial y}\right]=0.$ *Stationary Khokhlov–Zabolotskaya equation.*

2. $\dfrac{\partial^2 w}{\partial x^2}+\dfrac{\partial}{\partial y}\left(ae^{\beta w}\dfrac{\partial w}{\partial y}\right)=0.$ *Anisotropic heat/diffusion equation.*

3. $\dfrac{\partial}{\partial x}\left[f(w)\dfrac{\partial w}{\partial x}\right]+\dfrac{\partial}{\partial y}\left[g(w)\dfrac{\partial w}{\partial y}\right]=0.$ *Anisotropic heat/diffusion equation.*