



## 3. Nonlinear Elliptic Equations

### 3.1. Nonlinear Heat Equations of the Form $\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = f(w)$

1.  $\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = aw + bw^n.$
2.  $\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = aw^n + bw^{2n-1}.$
3.  $\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = ae^{\beta w}.$
4.  $\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = ae^{\beta w} + be^{2\beta w}.$
5.  $\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = \alpha w \ln(\beta w).$
6.  $\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = \alpha \sin(\beta w).$
7.  $\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = f(w).$

### 3.2. Heat Equations of the Form $\frac{\partial}{\partial x} \left[ f(x) \frac{\partial w}{\partial x} \right] + \frac{\partial}{\partial y} \left[ g(y) \frac{\partial w}{\partial y} \right] = f(w)$

1.  $\frac{\partial}{\partial x} \left( ax^n \frac{\partial w}{\partial x} \right) + \frac{\partial}{\partial y} \left( by^m \frac{\partial w}{\partial y} \right) = f(w).$  *Anisotropic heat/diffusion equation.*
2.  $a \frac{\partial^2 w}{\partial x^2} + \frac{\partial}{\partial y} \left( be^{\mu y} \frac{\partial w}{\partial y} \right) = f(w).$  *Anisotropic heat/diffusion equation.*
3.  $\frac{\partial}{\partial x} \left( ae^{\beta x} \frac{\partial w}{\partial x} \right) + \frac{\partial}{\partial y} \left( be^{\mu y} \frac{\partial w}{\partial y} \right) = f(w).$  *Anisotropic heat/diffusion equation.*
4.  $\frac{\partial}{\partial x} \left[ f(x) \frac{\partial w}{\partial x} \right] + \frac{\partial}{\partial y} \left[ g(y) \frac{\partial w}{\partial y} \right] = kw \ln w.$  *Anisotropic heat/diffusion equation.*

### 3.3. Heat Equations of the Form $\frac{\partial}{\partial x} \left[ f(w) \frac{\partial w}{\partial x} \right] + \frac{\partial}{\partial y} \left[ g(w) \frac{\partial w}{\partial y} \right] = h(w)$

1.  $\frac{\partial^2 w}{\partial x^2} + \frac{\partial}{\partial y} \left[ (\alpha w + \beta) \frac{\partial w}{\partial y} \right] = 0.$  *Stationary Khokhlov–Zabolotskaya equation.*

2.  $\frac{\partial^2 w}{\partial x^2} + \frac{\partial}{\partial y} \left( a e^{\beta w} \frac{\partial w}{\partial y} \right) = 0.$  *Anisotropic heat/diffusion equation.*

3.  $\frac{\partial}{\partial x} \left[ f(w) \frac{\partial w}{\partial x} \right] + \frac{\partial}{\partial y} \left[ g(w) \frac{\partial w}{\partial y} \right] = 0.$  *Anisotropic heat/diffusion equation.*

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The EqWorld website presents extensive information on solutions to various classes of ordinary differential equations, partial differential equations, integral equations, functional equations, and other mathematical equations.

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