4. $\dfrac{\partial w}{\partial t} = \dfrac{\partial^2 w}{\partial x^2} + aw + bw^m.$

1°. Traveling-wave solutions (the signs are chosen arbitrarily):

$$w(x,t) = \left[\pm\beta + C\exp(\lambda t \pm \mu x)\right]^{\frac{2}{1-m}},$$

where C is an arbitrary constant and $\beta = \sqrt{-\dfrac{b}{a}}$, $\lambda = \dfrac{a(1-m)(m+3)}{2(m+1)}$, $\mu = \sqrt{\dfrac{a(1-m)^2}{2(m+1)}}$.

2°. For $a = 0$, there is a self-similar solution of the form $w(x,t) = t^{1/(1-m)}U(z)$, where $z = xt^{-1/2}$.

References

Kaliappan, P., An exact solution for travelling waves of $u_t = Du_{xx} + u - u^k$, *Physica D*, Vol. 11, pp. 368–374, 1984.

Danilov, V. G., Maslov V. P., and Volosov, K. A., *Mathematical Modelling of Heat and Mass Transfer Processes*, Kluwer, Dordrecht, 1995.

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Nonlinear Partial Differential Equations*, Chapman & Hall/CRC, Boca Raton, 2004.

