6. $\dfrac{\partial w}{\partial t} = \dfrac{\partial^2 w}{\partial x^2} + aw \ln w.$

1°. Functional separable solutions:

$$w(x,t) = \exp\left(Ae^{at}x + \frac{A^2}{a}e^{2at} + Be^{at}\right),$$
$$w(x,t) = \exp\left[\tfrac{1}{2} - \tfrac{1}{4}a(x+A)^2 + Be^{at}\right],$$
$$w(x,t) = \exp\left[-\frac{a(x+A)^2}{4(1+Be^{-at})} + \frac{1}{2B}e^{at}\ln(1+Be^{-at}) + Ce^{at}\right],$$

where A, B, and C are arbitrary constants.

2°. Solution:

$$w(x,t) = \exp\left[Ae^{at} + f(x+bt)\right],$$

where A and b are arbitrary constants, and the function $f(\xi)$ is determined by the autonomous ordinary differential equation

$$f''_{\xi\xi} + (f'_\xi)^2 - bf'_\xi + af = 0.$$

References

Dorodnitsyn, V. A., On invariant solutions of the nonlinear heat equation with a source [in Russian], *Zhurn. vychisl. matem. i matem. fiziki*, Vol. 22, No. 6, pp. 1393–1400, 1982.

Samarskii, A. A., Galaktionov, V. A., Kurdyumov, S. P., and Mikhailov, A. P., *Blow-up in Problems for Quasilinear Parabolic Equations*, Walter de Gruyter, Berlin, 1995.

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Nonlinear Partial Differential Equations*, Chapman & Hall/CRC, Boca Raton, 2004.

