



6. $\frac{\partial w}{\partial t} = \frac{\partial^2 w}{\partial x^2} + aw \ln w.$

1°. Functional separable solutions:

$$w(x, t) = \exp\left(Ae^{at}x + \frac{A^2}{a}e^{2at} + Be^{at}\right),$$

$$w(x, t) = \exp\left[\frac{1}{2} - \frac{1}{4}a(x+A)^2 + Be^{at}\right],$$

$$w(x, t) = \exp\left[-\frac{a(x+A)^2}{4(1+Be^{-at})} + \frac{1}{2B}e^{at} \ln(1+Be^{-at}) + Ce^{at}\right],$$

where A , B , and C are arbitrary constants.

2°. Solution:

$$w(x, t) = \exp[Ae^{at} + f(x+bt)],$$

where A and b are arbitrary constants, and the function $f(\xi)$ is determined by the autonomous ordinary differential equation

$$f''_{\xi\xi} + (f'_{\xi})^2 - bf'_{\xi} + af = 0.$$

References

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