3. $\dfrac{\partial w}{\partial t} = a\dfrac{\partial}{\partial x}\left(w^m \dfrac{\partial w}{\partial x}\right) + bw^{m+1}.$

1°. Multiplicative separable solution ($a = b = 1$, $m > 0$):

$$w(x,t) = \begin{cases} \left[\dfrac{2(m+1)}{m(m+2)}\,\dfrac{\cos^2(\pi x/L)}{(t_0 - t)}\right]^{1/m} & \text{for } |x| \le \dfrac{L}{2},\\ 0 & \text{for } |x| > \dfrac{L}{2},\end{cases}$$

where $L = 2\pi(m+1)^{1/2}/m$. This solution describes a blow-up regime that exists on a limited time interval $t \in [0,\ t_0)$. The solution is localized in the interval $|x| < L/2$.

2°. Multiplicative separable solution:

$$w(x,t) = \left(\frac{Ae^{\mu x} + Be^{-\mu x} + D}{m\lambda t + C}\right)^{1/m},$$

$$B = \frac{\lambda^2(m+1)^2}{4b^2A(m+2)^2}, \quad D = -\frac{\lambda(m+1)}{b(m+2)}, \quad \mu = m\sqrt{-\frac{b}{a(m+1)}},$$

where A, C, and λ are arbitrary constants, $ab(m+1) < 0$.

3°. Functional separable solutions [it is assumed that $ab(m+1) < 0$]:

$$w(x,t) = \Bigl[F(t) + C_2|F(t)|^{\frac{m+2}{m+1}}e^{\lambda x}\Bigr]^{1/m}, \quad F(t) = \frac{1}{C_1 - bmt}, \quad \lambda = \pm m\sqrt{\frac{-b}{a(m+1)}},$$

where C_1 and C_2 are arbitrary constants.

4°. There are functional separable solutions of the following forms:

$$w(x,t) = \bigl[f(t) + g(t)(Ae^{\lambda x} + Be^{-\lambda x})\bigr]^{1/m}, \quad \lambda = m\sqrt{\frac{-b}{a(m+1)}};$$

$$w(x,t) = \bigl[f(t) + g(t)\cos(\lambda x + C)\bigr]^{1/m}, \quad \lambda = m\sqrt{\frac{b}{a(m+1)}},$$

where A, B, and C are arbitrary constants.

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