



Exact Solutions > Nonlinear Partial Differential Equations >
Second-Order Parabolic Partial Differential Equations > Heat Equation with a Exponential Nonlinearity

$$7. \quad \frac{\partial w}{\partial t} = a \frac{\partial}{\partial x} \left(e^{\lambda w} \frac{\partial w}{\partial x} \right).$$

Heat equation with a exponential nonlinearity.

1°. Solutions:

$$w(x, t) = \frac{2}{\lambda} \ln \left(\frac{\pm x + A}{\sqrt{B - 2at}} \right),$$
$$w(x, t) = \frac{1}{\lambda} \ln \frac{A + Bx - Cx^2}{D + 2aCt},$$

where A , B , C , and D are arbitrary constants.

2°. There are solutions of the following forms:

$$w(x, t) = F(z), \quad z = kx + \beta t \quad \text{traveling-wave solution;}$$

$$w(x, t) = G(\xi), \quad \xi = xt^{-1/2} \quad \text{self-similar solution;}$$

$$w(x, t) = H(\eta) + 2kt, \quad \eta = xe^{-k\lambda t};$$

$$w(x, t) = U(\zeta) - \lambda^{-1} \ln t, \quad \zeta = x + k \ln t,$$

where k and β are arbitrary constants.

References

- Ovsiannikov, L. V.**, Group properties of nonlinear heat equations [in Russian], *Doklady AN SSSR*, Vol. 125, No. 3, pp. 492–495, 1959.
- Ibragimov, N. H.** (Editor), *CRC Handbook of Lie Group Analysis of Differential Equations, Vol. 1, Symmetries, Exact Solutions and Conservation Laws*, CRC Press, Boca Raton, 1994.
- Polyanin, A. D. and Zaitsev, V. F.**, *Handbook of Nonlinear Partial Differential Equations*, Chapman & Hall/CRC, Boca Raton, 2004.

Heat Equation with a Exponential Nonlinearity